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The authors give the results of an analytical and experimental investigation of heat transfer between an axisymmetric jet and a plate normal to the flow for laminar boundary layer conditions.

The solution of a number of problems connected with drying, gas turbine combustion chamber calculations, drilling in hard rock and frozen soil, cutting through concrete, etc. often depends on jet heat transfer. Jet flow normal to a surface is particularly valuable for enhancing heat transfer, since the heat transfer coefficients are then several times greater [1-5], other things being equal, than for other methods of applying the heat transfer fluid, including longitudinal flow.

Let us examine the theoretical solution of the problem of heat transfer between a circular jet issuing from a nozzle of diameter d_0 at a velocity u_0 and a temperature t_0 , equal to the temperature t_∞ of the surrounding medium, and a flat disc of radius R with surface temperature t_w , normal to the axis of the jet. Let the distance from the nozzle to the disc be h , and let $t_w > t_\infty$ (there is no loss of generality if we assume $t_w < t_\infty$).

On contact with the heated surface the jet forms a boundary layer, which grows monotonically along the r axis. Let us denote the thickness of the thermal boundary layer by δ_t , and that of the dynamic boundary layer by δ .

Integration of the energy equation of the axisymmetric boundary layer, taking into account the continuity equation, yields the following integral energy equation of the boundary layer:

$$\frac{1}{r} \frac{d}{dr} r \int_0^t T u_r dy = -a \left(\frac{\partial t}{\partial y} \right)_{y=0}, \quad (1)$$

where $T = t - t_\infty$.

To solve (1) it is necessary to find the velocity and temperature distribution in the boundary layer, for which we use the Karman-Polhausen method, writing the temperature and velocity distributions in polynomial form

$$t = a_1 + b_1 y + c_1 y^2 + d_1 y^3, \quad u_r = a_2 + b_2 y + c_2 y^2 + d_2 y^3 \quad (2)$$

for the conditions:

a) temperature distribution

$$\text{when } y = 0 \quad t = t_w, \quad \frac{\partial^2 t}{\partial y^2} = 0,$$

$$\text{when } y = \delta_t \quad t = t_\infty, \quad \frac{\partial t}{\partial y} = 0;$$

b) velocity distribution

$$y = 0, \quad u_r = 0, \quad \frac{\partial^2 u_r}{\partial y^2} = 0^*,$$

$$y = \delta, \quad u_r = u_s, \quad \frac{\partial u}{\partial y} = 0.$$

Solving (2) in accordance with the boundary conditions gives the temperature and velocity distributions in the boundary layer:

$$T = T_w \left(1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \frac{y^3}{\delta_t^3} \right),$$

$$u_r = \frac{1}{2} u_s \frac{y}{\delta} \left[3 - \left(\frac{y}{\delta} \right)^2 \right],$$

*In jet theory it is usually assumed that $\partial p / \partial x = 0$.

where $T_w = t_w - t_\infty$.

In integrating (1) we must distinguish the cases $Pr > 1$ (dynamic boundary layer thickness greater than thermal) and $Pr < 1$ ($\delta_t > \delta$).

If $Pr > 1$, the energy equation takes the form:

$$\begin{aligned} & \frac{1}{r} \frac{d}{dr} r \int_0^{\delta_t} u_s T_w \left(1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \frac{y^3}{\delta_t^3} \right) \times \\ & \times \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \frac{y^3}{\delta^3} \right) dy = -a \left(\frac{\partial t}{\partial y} \right)_{y=0}. \end{aligned} \quad (3)$$

and if $Pr < 1$,

$$\begin{aligned} & \frac{1}{r} \frac{d}{dr} r \left[\int_0^{\delta} u_s T_w \left(1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \frac{y^3}{\delta_t^3} \right) \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \frac{y^3}{\delta^3} \right) dy + \right. \\ & \left. + \int_{\delta}^{\delta_t} u_s T_w \left(1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \frac{y^3}{\delta_t^3} \right) dy \right] = -a \left(\frac{\partial t}{\partial y} \right)_{y=0}. \end{aligned} \quad (3')$$

To solve the last equations, the thicknesses of the dynamic and thermal boundary layers must be related. For surfaces heated over their whole length, the well-known relation $\xi = Pr^{1/3}$ may be used.

Integrating (3) and (3') and introducing the Re number in terms of the thermal boundary layer thickness $Re_* = u_s \delta_t / \nu$, we have, after simple transformations,

$$r Re_* d(Re_* r) = \frac{10}{f(\xi)} \frac{u_s}{\nu} r^2 dr, \quad (4)$$

where $f(\xi) = \xi^2$ for $Pr > 1$ and

$$f(\xi) = (2.5 - 2.5\xi + \xi^2) \xi^3 \quad \text{for } Pr < 1.$$

From (4) we obtain the thermal boundary layer thickness

$$\delta_t = 4.475 f(Pr) \frac{\nu}{u_s r} \left(\int \frac{u_s}{\nu} r^2 dr + c \right)^{1/2}, \quad (5)$$

where

$$\begin{aligned} f(Pr) &= Pr^{-1/3} \quad \text{for } Pr > 1, \\ f(Pr) &= [(2.5 - 2.5 Pr^{1/3} + Pr^{2/3}) Pr]^{-1/2} \quad \text{for } Pr < 1. \end{aligned}$$

We shall assume that the velocity at the outer edge of the boundary layer u_s is proportional to the velocity on the jet axis.

In solving (5) two cases should be distinguished: when the sum of the distances from nozzle to plate is less than or equal to and when it is greater than the initial section of the jet, i. e.,

$$h + R \leq h_H = 6.2 d_0 \quad \text{and} \quad h + R > 6.2 d_0,$$

Starting from these two cases, we shall examine the following variants of the solution of (5).

The disc is located in the initial section of the jet, where the velocity along the axis is equal to the discharge velocity u_0 . Then $u_s = Ku_0$.

By analogy with axisymmetric flow over a disc in an infinite stream, for which $K = 2$ is obtained by solving the equation of motion in inviscid flow [7], we shall assume that in our case $K = 2$.

Substituting $u_s = 2u_0$ in (5) and integrating the radicand, we have

$$\delta_t = 2.24 f(Pr) \frac{\nu}{u_0 r} \left(\frac{2}{3} \frac{u_0 r^3}{\nu} + c \right)^{1/2}.$$

The constant of integration c is found from the condition $\delta_t = 0^*$ when $r = 0$. In our case $c = 0$, and we have finally

$$\delta_t = 1.83 f(\text{Pr}) r \text{Re}_r^{-1/2}.$$

The heat flux at the wall $q_w = -\lambda(\partial t/\partial y)_{y=0} = 3/2(\lambda)T_w/\delta_t$. On the other hand, $q_w = \alpha T_w$. Hence

$$\alpha = 3\lambda/2\delta_t. \quad (7)$$

Substituting the boundary layer thickness (6) in (7), we obtain the local heat transfer coefficient distribution

$$\alpha = 0.82 \frac{\lambda}{r} \frac{1}{f(\text{Pr})} \text{Re}_r^{1/2}, \quad (8)$$

or, in dimensionless form

$$\text{Nu}_r = 0.82 \frac{1}{f(\text{Pr})} \text{Re}_r^{1/2}. \quad (8')$$

The average heat transfer coefficient over the disc is found from the expression

$$\bar{\alpha} = \frac{1}{\pi R^2} \int_0^R 2\pi\alpha r dr.$$

Substituting the expression for α in the last equation and integrating, we obtain the average heat transfer coefficient

$$\bar{\alpha} = 1.09 \frac{\lambda}{R} \frac{1}{f(\text{Pr})} \text{Re}_R^{1/2},$$

or, in dimensionless form

$$\bar{\text{Nu}}_R = 1.09 \frac{1}{f(\text{Pr})} \text{Re}_R^{1/2}.$$

If D (diameter of disc) is taken as the characteristic dimension, the parametric equation becomes

$$\bar{\text{Nu}}_D = 1.54 \frac{1}{f(\text{Pr})} \text{Re}_D^{1/2}. \quad (9)$$

The disc is located in the main part of the jet. According to numerous experimental data, generalized in [8, 9], the velocity on the axis of an axisymmetric jet is $u_{\text{max}} = 6.2 d_0 u_0/h$, where the numerical coefficient takes into account the initial part of the jet.

As in the first case, we shall assume that $K = 2$.

The velocity at the outer edge of the boundary layer may therefore finally be written as: $u_s = 12.4 u_0 d_0/(h+r)$.

Substituting this expression in (5), we have

$$r \delta_t = \frac{4.475}{12.4} f(\text{Pr}) \frac{(h+r) \nu}{u_0 d_0} \left[\int \frac{12.4 u_0 d_0}{\nu(h+r)} r^2 dr + c \right]^{1/2}.$$

After integration, we obtain

$$r \delta_t = 0.361 f(\text{Pr}) \frac{h+r}{\text{Re}_{d_0}} \times \left\{ 12.4 \text{Re}_{d_0} h^2 \left[\frac{1}{2} (1+\bar{r})^2 - 2(1+\bar{r}) + \ln(1+\bar{r}) \right] + c \right\}^{1/2}.$$

The constant of integration is found from the condition that $\delta_t = 0$ when $\bar{r} = 0$. Hence $c = 12.4 \cdot 1.5 \text{Re}_{d_0} h^2$.

*In the first approximation the stagnation point region is neglected. If required, the stagnation point heat transfer coefficient may be calculated from the data of [10] and [11].

Simple transformations lead to an expression for the thermal boundary layer thickness

$$\delta_t = 1.27 f(\text{Pr}) \text{Re}_{d_0}^{-1/2} h \frac{1+\bar{r}}{\bar{r}} \left[\bar{r} \left(\frac{1}{2} \bar{r} - 1 \right) + \ln(1+\bar{r}) \right]^{1/2}. \quad (10)$$

The local heat transfer coefficient is obtained by substituting (10) in (7):

$$\alpha = 1.18 \frac{\lambda}{h} \frac{\text{Re}_{d_0}^{1/2}}{f(\text{Pr})} \frac{\bar{r}}{1+\bar{r}} \left[\bar{r} \left(\frac{1}{2} \bar{r} - 1 \right) + \ln(1+\bar{r}) \right]^{-1/2}, \quad (11)$$

or, in dimensionless form

$$\text{Nu}_r = 1.18 \frac{\text{Re}_{d_0}^{1/2}}{f(\text{Pr})} \frac{\bar{r}^2}{1+\bar{r}} \left[\bar{r} \left(\frac{1}{2} \bar{r} - 1 \right) + \ln(1+\bar{r}) \right]^{-1/2}. \quad (11')$$

Substituting (11) in the expression for the average heat transfer coefficient for the disc $\bar{\alpha}$ gives

$$\bar{\alpha} = 4.72 \frac{\lambda}{R} \frac{\text{Re}_{d_0}^{1/2}}{f(\text{Pr})} \frac{h}{R} \left[\frac{R}{h} \left(\frac{1}{2} \frac{R}{h} - 1 \right) + \ln \left(1 + \frac{R}{h} \right) \right]^{1/2},$$

or, in dimensionless form

$$\bar{\text{Nu}}_R = 4.72 \frac{\text{Re}_{d_0}^{1/2}}{f(\text{Pr})} \bar{R}^{-1} \left[\bar{R} \left(\frac{1}{2} \bar{R} - 1 \right) + \ln(1+\bar{R}) \right]^{1/2}. \quad (12)$$

Writing

$$\eta(\bar{R}) = \left[\bar{R} \left(\frac{1}{2} \bar{R} - 1 \right) + \ln(1+\bar{R}) \right]^{1/2} (\bar{R})^{-1},$$

we finally obtain

$$\bar{\text{Nu}}_R = 4.72 \text{Re}_{d_0}^{1/2} \eta(\bar{R}) [f(\text{Pr})]^{-1}. \quad (12')$$

In the range $\text{Pr} = 0.3 - 1.7$, i. e., in the region in which the majority of the experiments on jet heat transfer have been done, the value of the functions $f(\text{Pr})$ expressed by (6) may be approximated to within $\pm 3\%$ by the single relation

$$f(\text{Pr}) \approx \text{Pr}^{-1/3}.$$

Then formulas (8'), (9), (11'), and (12) assume a simpler form.

If the heat transfer surface is located in the initial section of the jet ($h + R \leq 6.2 d_0$), then:

local heat transfer coefficient

$$\text{Nu}_R = 0.82 \text{Pr}^{1/3} \text{Re}_r^{1/2}, \quad (13)$$

average heat transfer coefficient

$$\bar{\text{Nu}}_D = 1.54 \text{Pr}^{1/3} \text{Re}_D^{1/2}. \quad (14)$$

If the heat transfer surface is located in the main part of the jet ($h + R > 6.2 d_0$), then:

local heat transfer coefficient

$$\text{Nu}_r = 1.18 \text{Pr}^{1/3} \text{Re}_{d_0}^{1/2} \frac{\bar{r}^2}{1+\bar{r}} \left[\bar{r} \left(\frac{1}{2} \bar{r} - 1 \right) + \ln(1+\bar{r}) \right]^{-1/2}, \quad (15)$$

average heat transfer coefficient

$$\bar{\text{Nu}}_R = 4.72 \text{Pr}^{1/3} \frac{\text{Re}_{d_0}^{1/2}}{\bar{R}} \left[\bar{R} \left(\frac{1}{2} \bar{R} - 1 \right) + \ln(1+\bar{R}) \right]^{1/2}. \quad (16)$$

Experimental apparatus (Fig. 1) was set up to study the peculiarities of jet heat transfer.

Air is drawn from the room by a fan and passes successively through a control valve, an orifice plate meter, and a flexible hose to a circular nozzle, from which it issues as a jet.

The experimental heat-exchange elements were mounted normal to the jet with the axis collinear with the jet axis.

The heat-transfer surface was a copper disc 5-7 mm thick with its top surface polished to a mirror finish to minimize loss by radiation. The heat-exchange element had a main electrical heater, whose power was controlled by a voltage regulator in conjunction with a voltage stabilizer. The heater was designed to promote uniformity of disc surface temperature, to which the disc itself also contributed.

The copper disc and heater were placed in an aluminum cup to compensate for heat losses from the disc. The cup was heated from below by a guard heater connected with a power regulator and voltage stabilizer. The disc and the compensating cup were separated by an air gap 0.5 mm thick and held in a split textolite mount. To achieve a smooth flow over the heat-exchange element, the side walls of the mount were streamlined and carefully polished.

Copper-constantan thermocouples for measuring the disc surface temperature were located on the heat-transfer surface at radial intervals of 5-10 mm.

The compensating heat flux was controlled by means of differential thermocouples mounted on the surfaces of the disc and compensating cup and facing the air gap.

Tests were carried out with discs of two diameters: $D = 50$ and 150 mm.

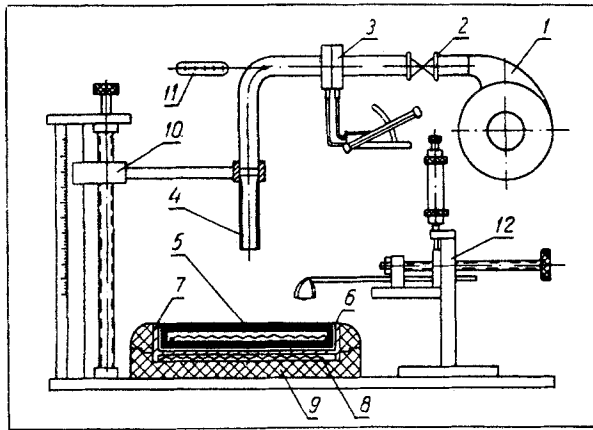


Fig. 1. Schematic of experimental setup:
 1 - fan; 2 - valve; 3 - orifice plate meter; 4 - nozzle;
 5 - heat-exchange element; 6 - main electric heater;
 7 - compensating cup; 8 - electric heater; 9 - body;
 10 - nozzle-positioning mechanism; 11 - thermocouple;
 12 - traversing unit.

In addition, the distance (h) from the nozzle to the surface of the disc was varied by means of a special nozzle-positioning device with a travel of 500 mm.

Circular nozzles, diameter $d_0 = 8, 10, 12, 20,$ and 30 mm, were used in the tests. The nozzle length was 40 diameters or more.

The mass flow of air through the nozzle was measured by the orifice plate meter, and the air temperature by a thermocouple. Moreover, the velocity field at the outlet was registered by means of a pressure probe.

The average heat transfer coefficient was determined from the formula

$$\bar{\alpha} = \frac{Q_{e.h.} - Q_r}{(t_w - t_\infty) F_D} \quad (17)$$

The amount of heat released by the main electric heater $Q_{e.h.}$ was computed from the readings of a precision wattmeter. The heat loss due to radiation Q_r was determined from the formula

$$Q_r = 4.9 \varepsilon \left[\left(\frac{T_w + 273}{100} \right)^4 - \left(\frac{T_\infty + 273}{100} \right)^4 \right],$$

where $\varepsilon = 0.025$ is the reduced emissivity of the heat transfer surface. Heat transfer by radiation is practically excluded by the mirror finish of the heat-transfer surface. For example, even for a surface temperature $t_w = 300^\circ\text{C}$ the radiation losses did not exceed 1% of the power supplied to the main heater.

The average temperature of the heat-transfer surface \bar{t}_w was computed from the formula

$$\bar{t}_w = [t_1 r_1^2 + t_2 (r_2^2 - r_1^2) + \dots + t_n (r_{n+1}^2 - r_n^2)] \left(\sum_1^n r_i^2 \right)^{-1}.$$

Besides the average heat transfer coefficients, the distribution of local heat transfer coefficients α was also determined. These were found graphically from the temperature fields in the boundary layer. These fields were measured with a 0.05-mm movable electrolytic thermocouple, controlled by a traversing unit, a micrometer being used for the vertical displacement. The temperature fields in the boundary layer were plotted every 5 mm over the entire diameter of the disc.

Comparison of average heat transfer coefficients calculated from (17) and from the distribution of the local coefficients showed good agreement. The maximum discrepancy did not exceed 15%. The experiments covered the following ranges of the main parameters: $Re_D = 8 \cdot 10^3 - 10^5$, $Re_{d_0} = 2 \cdot 10^3 - 2 \cdot 10^4$, $h/d_0 = 0.5 - 40$, $t_w = 80 - 300^\circ\text{C}$.

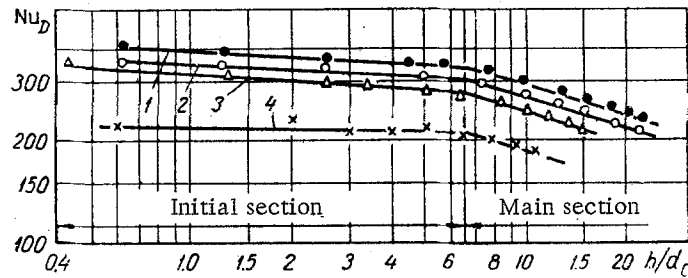


Fig. 2. Average heat transfer coefficient as a function of h/d_0 :
1 - $d_0 = 8$ mm; $u_0 = 29$ m/sec; 2 - 8, 22; 3 - 12, 17.5; 4 - 20, 11.1.

Analysis of the experimental data shows that two typical regions of laminar heat transfer may be clearly distinguished (Fig. 2):

1. When $h/d_0 \leq 6.2$, the heat transfer coefficient is practically independent of the distance h .
2. For values of $h/d_0 > 6.2$, the distance h has an appreciable influence on the heat transfer coefficient.

The boundary of these regions ($h/d_0 \approx 6.2$) coincides with the boundary between the initial and main hydrodynamic sections of the jet [8, 9].

Fig. 3. Comparison of experimental data on average heat transfer coefficients with the theoretical solution (14) for the region $h/d_0 \leq 6.2$ (initial section of jet): 1, 2, 3 - $d_0 = 8$ mm, 12, 20 (air, author's data); 4 - 10.7 (water, data of [4]); 5 - theory.

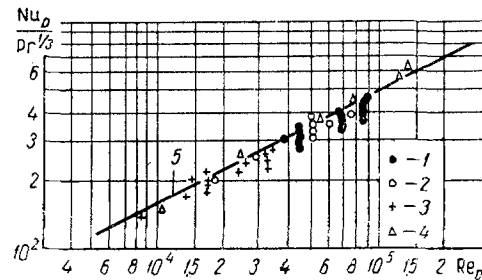


Figure 3 shows the reduced experimental results for average heat transfer coefficient in the first region $h/d_0 \leq 6.2$, i. e., when the surface is wholly in the initial section of the jet ($h + R \leq 6.2 d_0$). Also shown are the test data of [4] for heat transfer to a liquid jet (water).

It is clear from the figure that the theoretical formula

$$\overline{Nu}_D = 1.54 Pr^{1/2} Re_D^{1/2}$$

agrees quite satisfactorily with the experimental data and can be recommended for practical use. The average scatter of the test data does not exceed 10%.

The distribution of the local heat transfer coefficients is determined from relations (13) or (8').

In generalizing the test data, the physical parameters of the fluid were determined at the "average" temperature of the boundary layer

$$t_m = \frac{1}{2}(t_w + t_\infty).$$

The generalization of the test data given in Fig. 3 includes the following ranges of the basic parameters:

$$\frac{h}{d_0} = 0.5 - 6.2; Re_D = 8 \cdot 10^3 - 1.5 \cdot 10^5; Pr = 0.72 - 8.$$

It is interesting to compare how far the experiments on jet heat transfer in the region $h/d_0 \leq 6.2$ (initial section of jet) differ from those on heat transfer in transverse flow over plane surfaces in an infinite stream [10], for which the theoretical dependence of the average heat transfer has the form

$$\overline{Nu}_D = 1.52 Pr^{1/2} Re_D^{1/2}, \quad (18)$$

and the heat transfer in the vicinity of the forward stagnation point [11] the form:

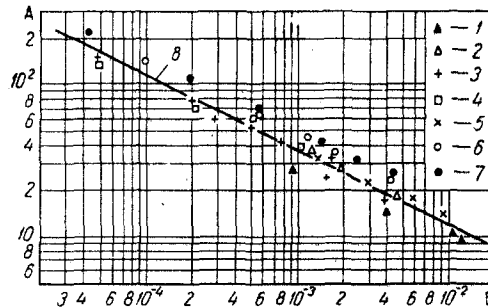
$$\overline{Nu}_D = 1.26 Pr^{1/2} Re_D^{1/2}. \quad (19)$$

Analysis of these relations shows that (14) and (18) practically coincide, while (19) is too low by 25%.

Fig. 4. Distribution of local heat transfer coefficients for the region $h/d_0 > 6.2$ (main section of jet):

$$A = Nu_r / Pr^{1/8} \frac{r^2}{1+r^2}; \quad B = \bar{r} (1/2 \bar{r} - 1) + \ln(1 + \bar{r});$$

1, 2, 3, 4 - author's tests at $h/d_0 = 6.3, 10, 12.5, 19$;
5, 6, 7 - tests of [6] at $h/d_0 = 6, 16, 24$; 8 - theory.



For the main section of the jet, $h/d_0 > 6.2$, Fig. 4 shows the authors' experimental results, together with those of [6], for the distribution of local heat transfer coefficients over the disc radius. The theoretical relation (15) is also given.

Comparison of the experimental data on local heat transfer with the theoretical solution shows good agreement. The average scatter of the test data does not exceed 12%.

Figure 5 gives the reduced test data on the average heat transfer coefficient for the same region ($h + R > 6.2 d_0$).

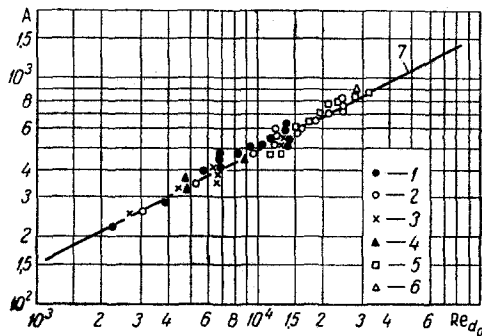


Fig. 5. Average heat transfer coefficient for the region $h/d_0 > 6.2$:

$$A = \overline{Nu} \bar{R} / Pr^{1/3} [\bar{R} (1/2 \bar{R} - 1) + \ln(1 + \bar{r})];$$

1, 2, 3, 4 - $d_0 = 8$ mm; 12, 20, 30 (author's data); 5 - 16.5 (experiments of [2]); 6 - 6.35 (experiments of [6]); 7 - theory.

In addition to the authors' data, Fig. 5 generalizes the experiments of [2] and [6]. It is evident from the figure that (16) shows completely satisfactory agreement with the results of experiments.

As for the initial section of the jet, the physical parameters of the fluid were again determined for the "average" boundary layer temperature t_{m1} .

The generalized test data given in Figs. 4 and 5 cover the following ranges of variation of the main parameters: $h/d_0 = 6.3-40$; $Re_{d_0} = 2000-28\ 000$; $\bar{R} = 0.02-0.5$.

NOTATION

d_0 - nozzle diameter; R and D - disc radius and diameter, respectively; h - distance from nozzle to heat-transfer surface; r - radial coordinate of disc; h/d_0 - relative distance from nozzle to heat-transfer surface; $\bar{r} = r/h$ - relative radial coordinate; $\bar{R} = R/h$ - dimensionless radius of disc; t_w - surface temperature; t_∞ - temperature outside boundary layer; u_s - velocity at outer edge of boundary layer; u_0 - discharge velocity; α - local heat transfer coefficient; $\bar{\alpha}$ - average heat transfer coefficient over disc; Nu - local Nusselt number; \overline{Nu} - average Nusselt number; Nu_r , Nu_R and Nu_D - Nusselt numbers referred respectively to r , R , and D ; $Re_r = u_0 r / \nu$, Re_{d_0} , Re_R , Re_D - Reynolds number referred respectively to r , d_0 , R , and D .

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